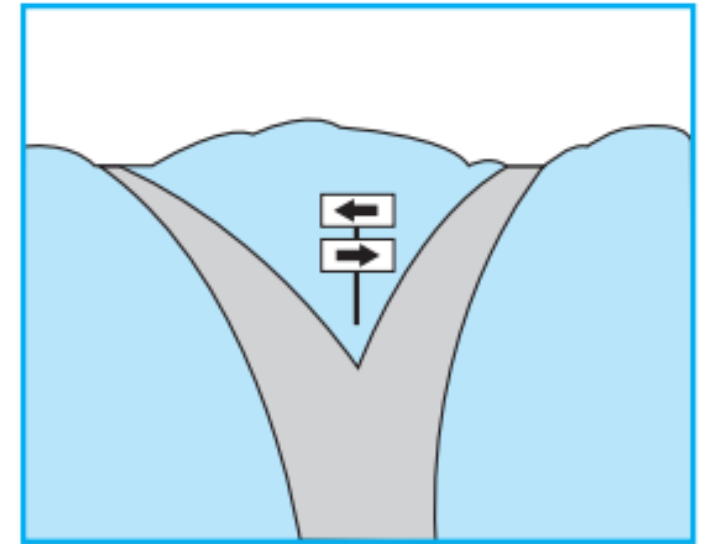
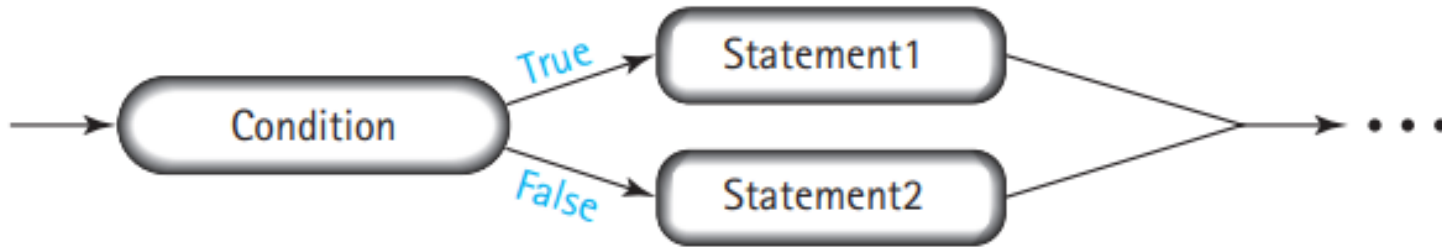


# Introduction to Boolean Logic

# What Kinds of Instructions Do We Use?

**SELECTION** (also called *branch* or *decision*)

IF condition THEN statement1 ELSE statement2



# What is Condition (based on which we do selection)?

- Boolean logic
  - George Boole is a contemporary of Charles Babbage, the “forefather of the information age.”
- Truth tables
  - OR
  - AND
  - NOT

# Introduction

- In the latter part of the nineteenth century, George Boole incensed philosophers and mathematicians alike when he suggested that logical thought could be represented through mathematical equations.
  - *How dare anyone suggest that human thought could be encapsulated and manipulated like an algebraic formula?*
- Computers, as we know them today, are implementations of Boole's *Laws of Thought*.
  - John Atanasoff and Claude Shannon were among the first to see this connection.

# Introduction

- In the middle of the twentieth century, computers were commonly known as “thinking machines” and “electronic brains.”
  - Many people were fearful of them.
- Nowadays, we rarely ponder the relationship between electronic digital computers and human logic. Computers are accepted as part of our lives.
  - Many people, however, are still fearful of them.
- In this chapter, you will learn the simplicity that constitutes the essence of the machine.

# Introduction

- Boolean algebra is a mathematical system for the manipulation of variables that can have one of two values.
  - In formal logic, these values are “true” and “false.”
  - In digital systems, these values are
    - “on” and “off”
    - 1 and 0
    - “high” and “low”
- Boolean expressions are created by performing operations on Boolean variables.
  - Common Boolean operators include AND, OR, and NOT.

# Boolean Variables

- In Boolean algebra, there are only two constants: True and False
  - ➔ Boolean variables can have one of the two values: TRUE or FALSE
- Boolean variables are variables that store values that are Boolean constants.

# What Does it Mean that Something is TRUE/ FALSE?

Are the statements below TRUE or FALSE:

- You are a CUNY Student.
- You are a Linguistics major.
- It is snowing outside.
- It is raining outside.
- It is sunny outside
- You can vote in the US 2020 election.



# Selection based on Condition

If the weather is nice, I will walk home.

Otherwise I will take CitiBike.

In either case I will be at home by 9pm.

# Selection based on Condition

If the weather is nice, I will walk home.

Otherwise I will take CitiBike.

In either case I will be at home by 9pm.

The weather is nice.  
(TRUE or FALSE)



I will walk home.

I will take CitiBike.

# Selection based on Condition

If the weather is nice, I will walk home.

Otherwise I will take CitiBike.

In either case I will be at home by 9pm.

condition

The weather is nice.  
(TRUE or FALSE)

selection based  
on condition

TRUE

FALSE

I will walk home.

I will take CitiBike.

actions  
according  
to selection

I will be home by 9pm

if (the weather is nice):  
    I will walk home  
else:  
    I will take CitiBike

if (condition is **TRUE**):  
    action 1  
else:  
    action 2

# if condition is **TRUE**  
  
# if condition is **FALSE**

# Sometimes you need to take into consideration more than one condition.

Can you vote in the US elections?

You must be a US citizen

**AND**

You must be at least 18 years old.

BOTH of these conditions must be TRUE for you to be able to vote.

# Selection based on Condition

- Voting

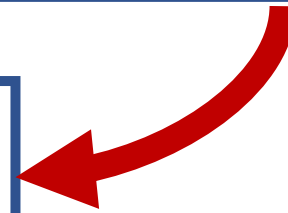
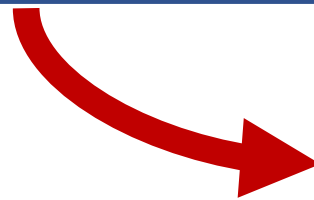
You are a US citizen AND You are at least 18 y.o.

TRUE

FALSE

You can vote.

You cannot vote.



# Sometimes you need to take into consideration more than one condition.

Can you vote in the US elections?

You must be a US citizen

**AND**

You must be at least 18 years old

**AND**

You must register for voting

ALL THREE of these conditions must be TRUE for you to be able to vote.

# Sometimes you need to take into consideration more than one condition.

Can you vote in the US elections?

You must be a US citizen

**AND**

You must be at least 18 years old

**AND**

You must register for voting

You are US citizen of at least 18 y.o

**AND**

You must register for voting

ALL THREE of these conditions must be TRUE for you to be able to vote.

OR, you can reformulate three conditions as two (remember, how you use parenthesis in math?)



# Voting

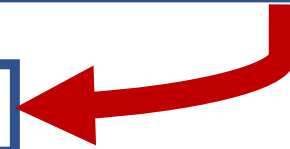
You are a US citizen AND You are at least 18 y.o. AND You have registered to vote

TRUE

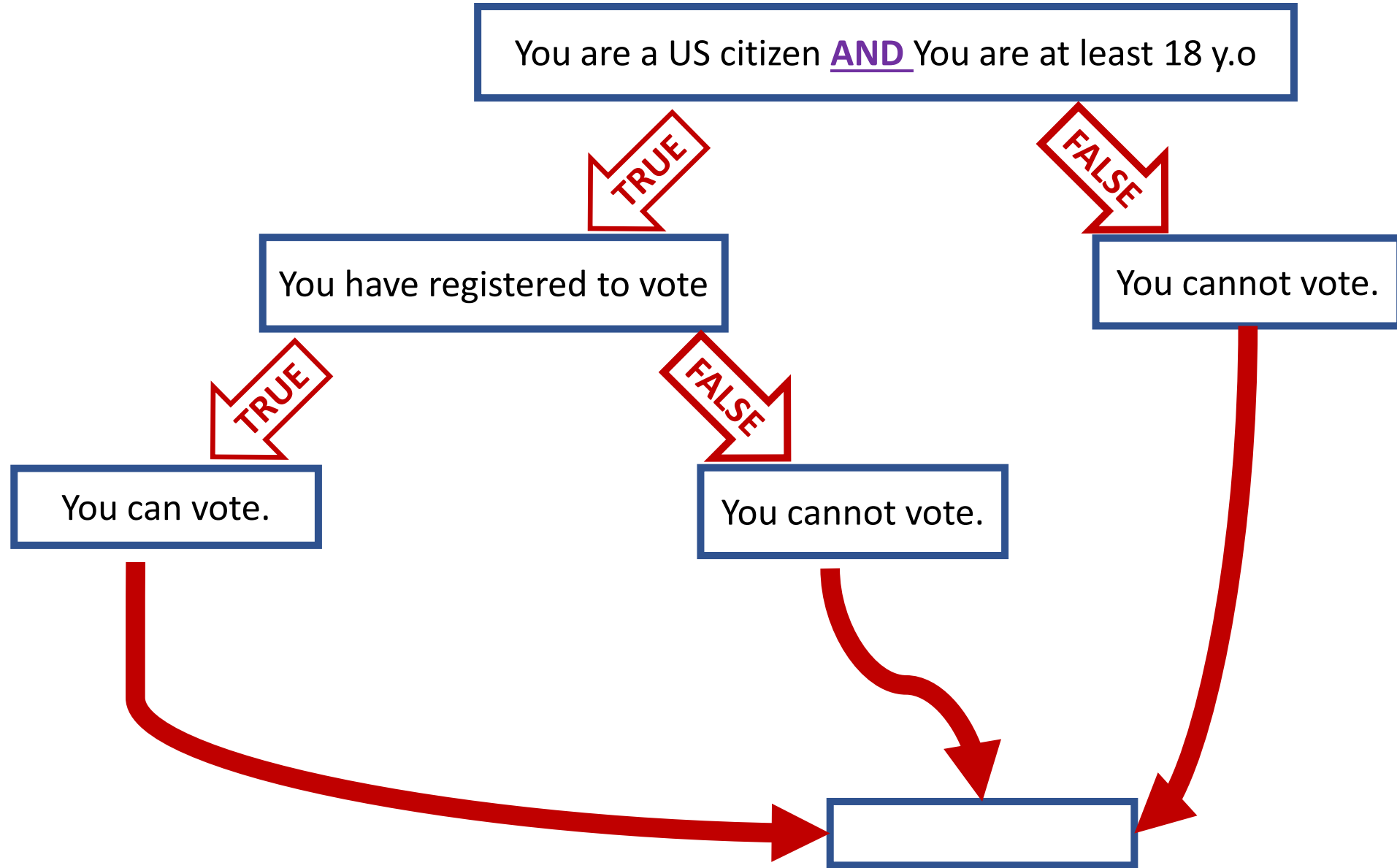
FALSE

You can vote.

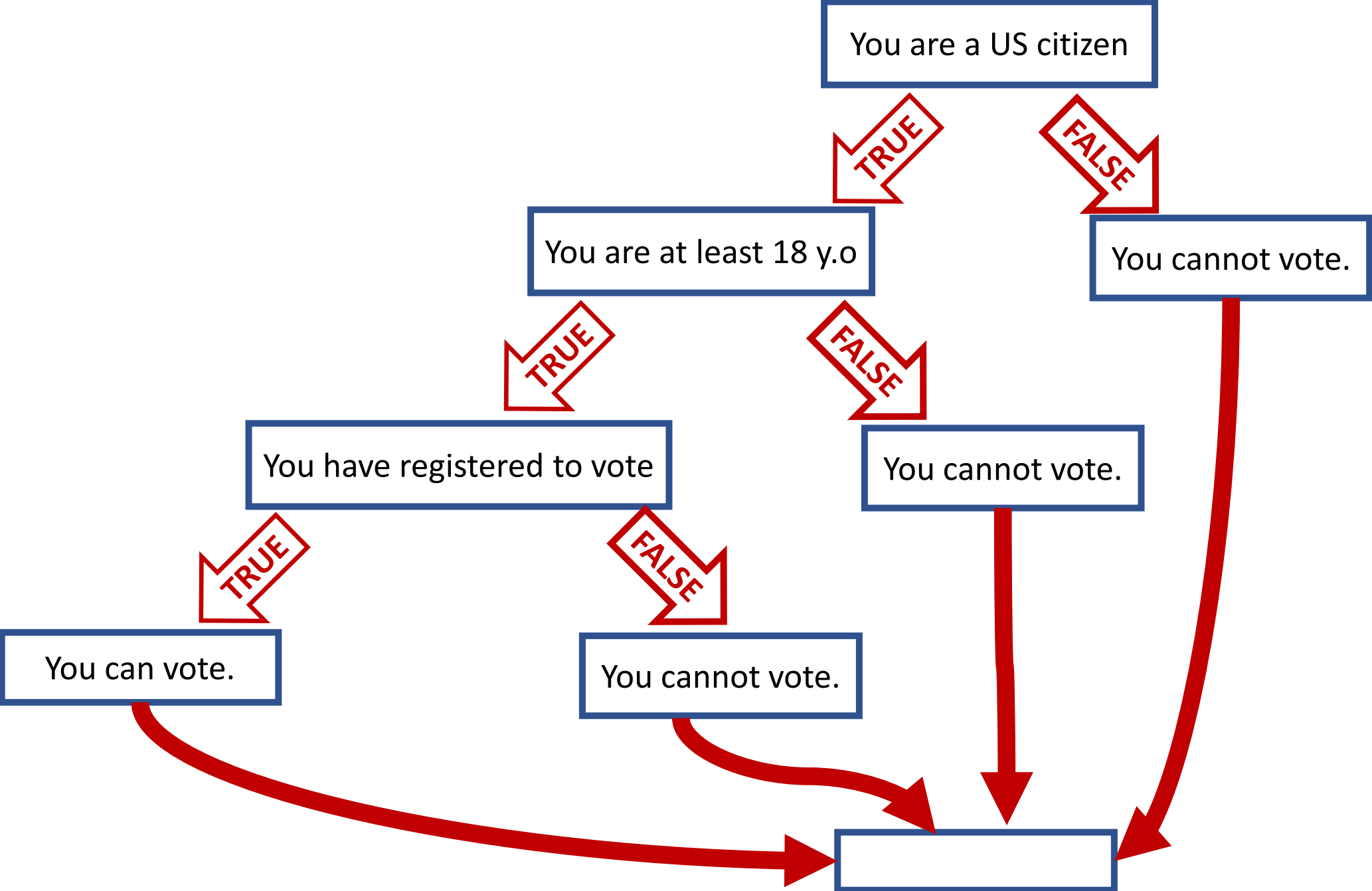
You cannot vote.



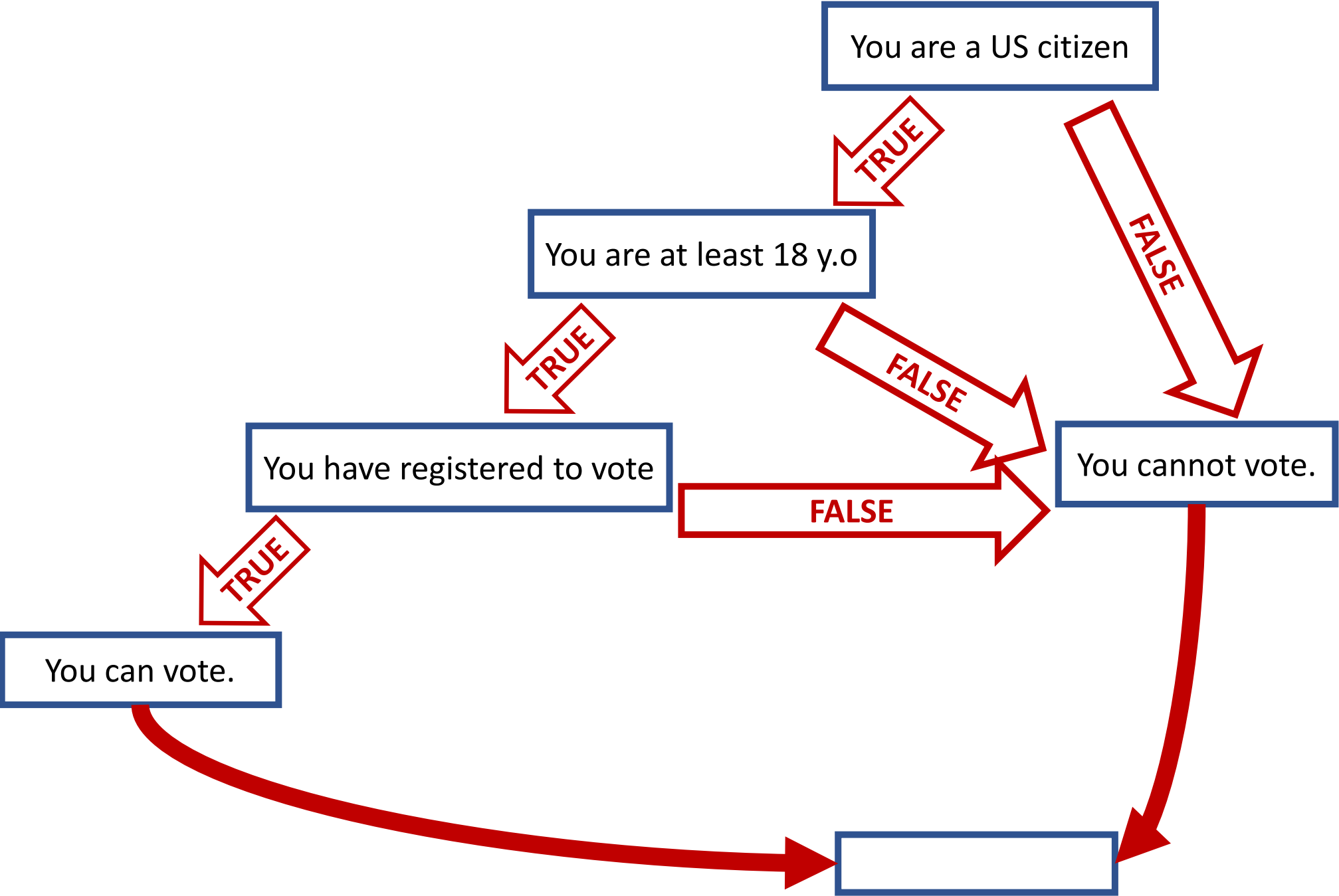
# Voting



Voting



Voting



# TRUTH table: AND

<b>You are a US citizen</b> <u>A</u>	<b>You are at least 18 y.o.</b> <u>B</u>	<b>You can vote</b> <u>A and B</u>
FALSE	FALSE	FALSE
FALSE	TRUE	FALSE
TRUE	FALSE	FALSE
TRUE	TRUE	TRUE

# TRUTH table: **AND** (TRUE = 1; FALSE = 0)

<b>You are a US citizen</b> <u>A</u>	<b>You are at least 18 y.o.</b> <u>B</u>	<b>You can vote</b> <u>A and B</u>
<b>0</b>	<b>0</b>	<b>0</b>
<b>0</b>	<b>1</b>	<b>0</b>
<b>1</b>	<b>0</b>	<b>0</b>
<b>1</b>	<b>1</b>	<b>1</b>

2 variables; 2 values for each of the variables → 4 possible combinations

# TRUTH table: **AND** (TRUE = 1; FALSE = 0)

<b>You have registered to vote</b>	<b>You are a US citizen</b>	<b>You are at least 18 y.o.</b>	<b>You can vote</b>
<b>0</b>	<b>0</b>	<b>0</b>	
<b>0</b>	<b>0</b>	<b>1</b>	
<b>0</b>	<b>1</b>	<b>0</b>	
<b>0</b>	<b>1</b>	<b>1</b>	
<b>1</b>	<b>0</b>	<b>0</b>	
<b>1</b>	<b>0</b>	<b>1</b>	
<b>1</b>	<b>1</b>	<b>0</b>	
<b>1</b>	<b>1</b>	<b>1</b>	

3 variables; 2 values for each of the variables → 8 possible combinations ( $2^3=8$ )

# How to computer the TRUTH table with more than two variables?

- All the decisions must be binary.
- Do step by step computation.



# TRUTH tables: **AND** (TRUE = 1; FALSE = 0)

<b>You have registered to vote</b> <b>A</b>	<b>You are a US citizen</b> <b>B</b>	<b>A and B</b>	<b>You are at least 18 y.o.</b>	<b>You can vote</b>
<b>0</b>	<b>0</b>		<b>0</b>	
<b>0</b>	<b>0</b>		<b>1</b>	
<b>0</b>	<b>1</b>		<b>0</b>	
<b>0</b>	<b>1</b>		<b>1</b>	
<b>1</b>	<b>0</b>		<b>0</b>	
<b>1</b>	<b>0</b>		<b>1</b>	
<b>1</b>	<b>1</b>		<b>0</b>	
<b>1</b>	<b>1</b>		<b>1</b>	

# TRUTH tables: **AND** (TRUE = 1; FALSE = 0)

<b>You have registered to vote</b> <b>A</b>	<b>You are a US citizen</b> <b>B</b>	<b>A and B</b>	<b>You are at least 18 y.o.</b>	<b>You can vote</b>
<b>0</b>	<b>0</b>	<b>0</b>	<b>0</b>	
<b>0</b>	<b>0</b>	<b>0</b>	<b>1</b>	
<b>0</b>	<b>1</b>	<b>0</b>	<b>0</b>	
<b>0</b>	<b>1</b>	<b>0</b>	<b>1</b>	
<b>1</b>	<b>0</b>	<b>0</b>	<b>0</b>	
<b>1</b>	<b>0</b>	<b>0</b>	<b>1</b>	
<b>1</b>	<b>1</b>	<b>1</b>	<b>0</b>	
<b>1</b>	<b>1</b>	<b>1</b>	<b>1</b>	

# TRUTH tables: TRUE = 1; FALSE = 0

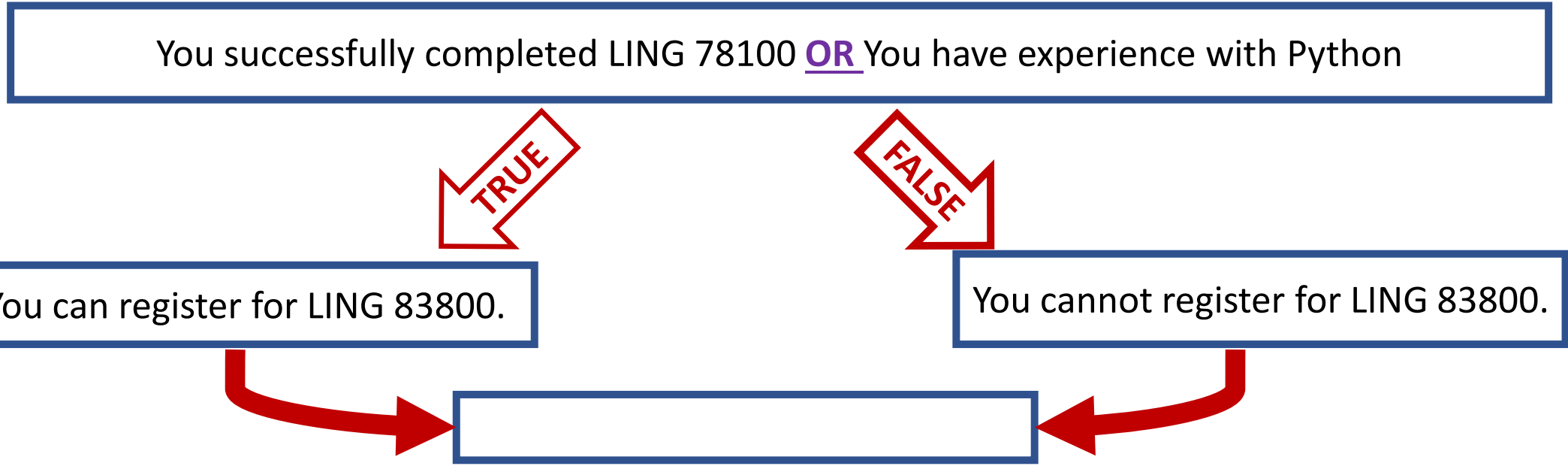
<b>You have registered to vote</b> A	<b>You are a US citizen</b> B	<b>A and B</b> B	<b>You are at least 18 y.o.</b>	<b>You can vote</b>
0	0	0	0	
0	0	0	1	
0	1	0	0	
0	1	0	1	
1	0	0	0	
1	0	0	1	
1	1	1	0	
1	1	1	1	

# TRUTH tables: TRUE = 1; FALSE = 0

<b>You have registered to vote</b> <b>A</b>	<b>You are a US citizen</b> <b>B</b>	<b>A and B</b> <b>B</b>	<b>You are at least 18 y.o.</b>	<b>You can vote</b>
<b>0</b>	<b>0</b>	<b>0</b>	<b>0</b>	<b>0</b>
<b>0</b>	<b>0</b>	<b>0</b>	<b>1</b>	<b>0</b>
<b>0</b>	<b>1</b>	<b>0</b>	<b>0</b>	<b>0</b>
<b>0</b>	<b>1</b>	<b>0</b>	<b>1</b>	<b>0</b>
<b>1</b>	<b>0</b>	<b>0</b>	<b>0</b>	<b>0</b>
<b>1</b>	<b>0</b>	<b>0</b>	<b>1</b>	<b>0</b>
<b>1</b>	<b>1</b>	<b>1</b>	<b>0</b>	<b>0</b>
<b>1</b>	<b>1</b>	<b>1</b>	<b>1</b>	<b>1</b>

LING 83800 registration

OR



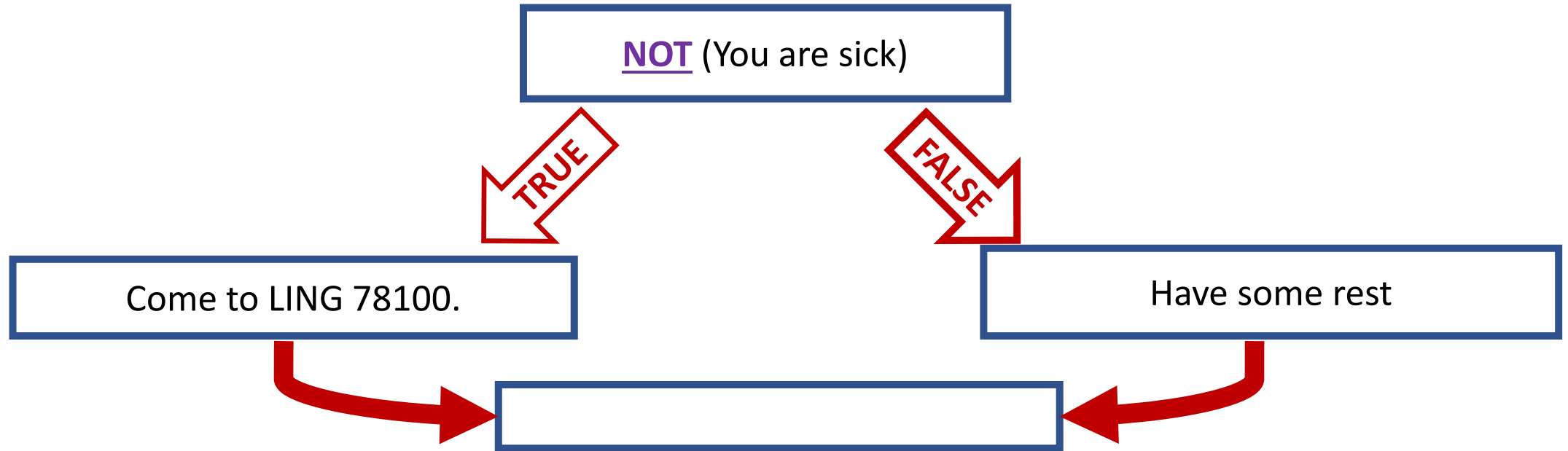
# TRUTH table: **OR** (TRUE = 1; FALSE = 0)

<u>A</u>	<u>B</u>	<u>A and B</u>
0	0	0
0	1	1
1	0	1
1	1	1

2 variables; 2 values for each of the variables → 4 possible combinations

If you are not sick you should come to LING78100.

NOT



TRUTH table: **NOT** (TRUE = 1; FALSE = 0)

<u>You are sick</u>	<u>NOT(You are sick)</u>
0	1
1	0

<u>A</u>	<u>NOT(A)</u>
0	1
1	0

1 variables; 2 values for each of the variables → 2 possible combinations



# Order of logical operations

1. NOT
2. AND
3. OR

Use parenthesis for easier reading.

# Exercises: create truth tables

*(all variables are Boolean variables)*

- $A \text{ and } B \text{ or } C \rightarrow (A \text{ and } B) \text{ or } C$
- $\text{not } A \text{ and } B \text{ or } C \rightarrow ((\text{not } A) \text{ and } B) \text{ or } C$
- $\text{not } (A \text{ and } B) \text{ or } C \rightarrow (\text{not } (A \text{ and } B)) \text{ or } C$
- $\text{not } A \text{ or } B \rightarrow (\text{not } A) \text{ or } B$
- $\text{not } (A \text{ or } B)$